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STRENGTH TESTS OF THIN-WALLED DURALUMIN

CYLINDERS OF ELLIPTIC SECTION

By Eugene E. Lundquist and Walter F. Burke
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SUMMARY

This report is the fifth of a series presenting the results of strength tests of thin-walled cylinders and truncated cones of circular and elliptic section; it includes the results obtained from torsion tests on 30 cylinders, pure bending tests on 30 cylinders, and combined transverse shear and bending tests on 60 cylinders. All the cylinders tested were of elliptic section with the ends clamped to rigid bulkheads. In the pure bending and combined transverse shear and bending tests the loads were applied in the plane of the major axis.

The results of the tests on elliptic cylinders are correlated with the results of corresponding tests on circular cylinders and are presented in charts suitable for use in design.

INTRODUCTION

As part of an investigation of the strength of stressed-skin, or monocoque, structures for aircraft, the National Advisory Committee for Aeronautics in cooperation with the Army Air Corps; the Bureau of Aeronautics, Navy Department; the National Bureau of Standards; and the Bureau of Air Commerce has made an extensive series of tests on thin-walled duralumin cylinders and truncated cones of circular and elliptic section. In these tests, the absolute and relative dimensions of the specimens were varied to study the types of failure and to establish useful quantitative data in the following loading conditions: torsion, compression, bending, and combined loading.

The first four reports of this series (references 1 to 4, inclusive) present the results obtained in the tests of thin-walled cylinders of circular section. This report

presents the results obtained in tests of thin-walled cyl-
inders of elliptic section. In the tests of the elliptic
cylinders the loading conditions were torsion, pure bending,
and combined transverse shear and bending. In the latter
two loading conditions the loads were applied in the plane
of the major axis.

MATERIAL

The duralumin (Al. Co. of Am. 17ST) used in these
tests was obtained from the manufacturer in sheet form
with nominal thicknesses of 0.011, 0.016, and 0.022 inch.
The properties of this material as determined by the
National Bureau of Standards from specimens selected at
random are given in references 1 and 2. As all the test
cylinders failed by elastic buckling of the walls at
stresses considerably below the yield-point stress, the
modulus of elasticity E, which was substantially constant
for all sheet thicknesses, is the only important property
of the material that need be considered. For all cylin-
ders an average value of E (10.4 × 10⁶ pounds per square
inch) was used in the analysis of the results.

SPECIMENS

The test specimens were right elliptic cylinders with
a semimajor axis of 7.5 inches and semiminor axes of 6.0
and 4.5 inches. The lengths varied from 3.75 to 22.5
inches. The cylinders were constructed in the following
manner: First, a duralumin sheet was cut to the dimensions
of the developed surface. The sheet was then wrapped about
and clamped to end bulkheads. (See figs. 1, 2, and 3.)
With the cylinder thus assembled, a butt strap 1 inch wide
and of the same thickness as the sheet was fitted, drilled,
and bolted in place to close the seam, which was always lo-
cated at one end of the minor axis. In the assembly of the
specimen, an effort was made to avoid having either a
looseness of the skin (soft spots) or wrinkles in the walls
when finally constructed. Actually this objective could
not be attained at the ends of the minor axis where there
was always a slight looseness of the skin which was more
pronounced in the thinner sheets than in the thicker ones.

The end bulkheads, to which the loads were applied,
were each constructed of two steel plates one-quarter inch

thick separated by a plywood core 1-1/2 inches thick.
These parts were bolted together and machined to the spec-
ified outside dimensions. Steel bands approximately one-
quarter inch thick were used to clamp the duralumin sheet
to the bulkheads. These bands were machined to the same
dimensions as the bulkheads.

APPARATUS AND METHOD

The thickness of each sheet was measured to an esti-
mated precision of ±0.0003 inch at a large number of sta-
tions by means of a dial gage mounted in a special jig.
In general, the variation in thickness throughout a given
sheet was not more than 2 percent of the average thickness.
The average thicknesses of the sheets were used in all
calculations of stress and radius-thickness ratio.

The loads were applied to the elliptic cylinders with
the same apparatus used in the corresponding tests on cir-
cular cylinders. Descriptions and photographs of the ap-
paratus used in the torsion, pure bending, and combined
transverse shear and bending tests are given in references
1, 3, and 4.

DISCUSSION OF RESULTS

The results of the tests on the elliptic cylinders
are presented in tables I, II, and III as well as in cer-
tain figures discussed later.

Torsion.- The torque loads were applied in increments
of about 1 percent of the estimated load at failure. Af-
ter a few tests, however, the first increment of load was
increased to about half the estimated load at first wrin-
kle. At first wrinkling one or more diagonal wrinkles be-
gan to form at the ends of the minor axis. With an in-
crease in load these wrinkles grew steadily in size and
number, spreading toward the ends of the major axis until
failure occurred by a sudden increase in deformation and
the formation of wrinkles in the complete circumference.
(See fig. 1.)

Prior to wrinkling it seems reasonable to assume that
the conditions of stress within the walls of the cylinder
are in accord with the membrane analogy (reference 5).

After wrinkling has occurred, however, the conditions of stress are doubtful and probably vary with the degree of buckling.

By the membrane analogy

$$S = \frac{T}{2At} \tag{1}$$

where

S,   shearing stress in the skin.

T,   torque applied.

$A = \pi ab$,   area of the ellipse.

t,   thickness of the sheet.

The shearing stress as given by equation (1) was calculated for each test cylinder at the conditions of first wrinkle and ultimate load. These stresses are used in plotting the experimental points in figure 4, which is the same type of chart used by Donnell in reference 6 for presenting the results of torsion tests on thin-walled cylinders of circular section. It will be noted that the experimental points for ultimate load plot between the theoretical curves and those recommended for design. As the data from torsion tests of circular cylinders also plot between these same curves (reference 6, fig. 1), it may be concluded that the shearing stress at ultimate load for an elliptic cylinder is the same as that for the circumscribed circular cylinder of the same sheet thickness and length. Actually the membrane analogy does not give a true picture of the stress condition at ultimate load but gives a shearing stress which lies between that at the ends of the semimajor and semiminor axes. It must therefore be regarded as an average shearing stress for the elliptic cylinder and analogous to the modulus of rupture in beams.

As the curvature of the sheet varies from a minimum at the ends of the minor axis to a maximum at the ends of the major axis, it is natural to expect that wrinkling prior to failure will occur in an elliptic cylinder subjected to torsion. In a perfectly constructed cylinder it is not likely that first wrinkling would occur until the shearing stress reaches or exceeds S_o, the shearing stress at failure for a circular cylinder of the same sheet thickness and length but with a radius equal to the

radius of curvature at the ends of the minor axis. The effect of imperfections, which are always present, is to cause a premature wrinkling. Consequently, the shearing stress at first wrinkle S_W was divided by S_o and the ratio plotted as a percentage in figure 5. With the exception of one point, all of the results plot below 100 percent. This figure is analogous to figure 10 of reference 1.

When calculating the value of S_o for the points in figure 5, the equation given in reference 1 was used because it is more representative of the technic used in this series of tests. For general design, the equations given in reference 6 are recommended.

Pure bending.- In the pure bending tests the loads were applied in the plane of the major axis of the ellipse. As in the torsion tests, the loads were applied in increments of about 1 percent of the estimated load at failure. In the case of the 0.8 ellipse $\left(\dfrac{b}{a} = 0.8\right)$, wrinkling did not usually occur prior to failure. Failure is defined as the complete collapse of the compression side of the cylinders with the formation of diamond-shaped wrinkles. (See fig. 2.) In the case of the 0.6 ellipse $\left(\dfrac{b}{a} = 0.6\right)$, preliminary wrinkles usually formed prior to failure. These wrinkles always appeared at about two thirds of the distance from the neutral axis to the extreme fiber.

The reason for preliminary wrinkles forming prior to failure in the 0.6 ellipse and not in the 0.8 ellipse is shown qualitatively in figure 6. The critical compressive stress for any element is approximately proportional to the curvature, and hence to the abscissa, of the solid curves in figure 6. In a cylinder subjected to pure bending the actual stress varies linearly from the neutral axis to the extreme fiber. When the loading has reached such a point that the actual stress is approximately that shown by the dotted lines, first wrinkling is likely to occur at the point where the solid and dotted curves touch. For the 0.6 ellipse, the elements at the extreme fiber are capable of resisting a much higher stress than indicated by the dotted line with the result that failure is delayed until a much higher load is reached. For the 0.8 ellipse failure tends to occur at or soon after first wrinkling because the elements on the extreme fiber and for a considerable distance toward the neutral axis are not capable of resisting any large increase in stress.

In the presentation of the results of the pure bend-
ing tests, it has been assumed that the ordinary theory
of bending applies and that the stress on the extreme com-
pression fiber at failure is given by the equation

$$S_b = \frac{Mc}{I} = \frac{M}{k_b\,a^2\,t} \tag{2}$$

where M is the bending moment applied in the plane of
the major axis of the ellipse

a, the semimajor axis of the ellipse

and k_b, a nondimensional coefficient that varies with
the eccentricity of the ellipse. (See fig. 7.)

The stress on the extreme fiber at failure as given
by equation (2) is comparable with the corresponding stress
calculated for the pure bending tests on circular cylin-
ders (reference 3) and, for comparison, is plotted in a
like manner in figure 8.   Inspection of this figure shows
that the stress on the extreme fiber at failure increases
with decrease in the ratio b/a.   The increased strength
for the more eccentric ellipse is obviously caused by the
greater stability of the more sharply curved portion at
the end of the major axis where the fibers are more highly
stressed.   It will be noted in figure 8 that the results
of the tests of elliptic cylinders scatter almost as widely
as do the results of the tests of circular cylinders.   The
designer in using these data may therefore choose strength
values more or less conservatively, as desired.

Combined transverse shear and bending.- The loads in
the transverse shear and bending tests were applied in the
plane of the major axis of the ellipse in increments of
about 1 percent of the estimated load at failure.   When
the position of the transverse load was such that the ratio
of moment to shear was small, failure occurred by shear.
Prior to failure, diagonal shear wrinkles formed on the
sides of the cylinder.   With an increase in load, these
wrinkles grew steadily in size until failure occurred by a
collapse of the outermost compression elements.   When the
position of the transverse load was such that the ratio of
moment to shear was large, failure occurred by bending in
the same manner as in the pure bending tests.   When the
ratio of moment to shear had an intermediate value, fail-
ure occurred by a combination of shear and bending.   (See
photographs of cylinders after failure, fig. 3.)

The variation of the bending stress at failure with the ratio of moment to shear is studied for each of the following groups of cylinders tested.

Group	$\frac{b}{a}$	$\frac{length}{a}$	$\frac{a}{t}$	Nominal sheet thickness
				Inch
1	0.8	0.50	630 - 682	0.011
2	.6	.50	625 - 688	.011
3	.8	1.00	658 - 707	.011
4	.6	1.00	658 - 721	.011
5	.6	2.00	630 - 682	.011
6	.8	2.00	630 - 682	.011

For each group, the results of the tests are presented in a manner similar to that used for the corresponding tests on circular cylinders (reference 4). In figure 9 the nondimensional parameter $\frac{M}{r'V}$ represents the loading and stress conditions and is therefore analogous to $\frac{M}{rV}$ in reference 4. The value of r' was so chosen that for the elliptic cylinder

$$\frac{M}{r'V} = \frac{f_b}{f_v} \tag{3}$$

where f_b and f_v are the bending stress on the extreme fiber and the shearing stress at the neutral axis respectively as given by the following equations derived by the ordinary theory of bending

$$f_b = \frac{M}{k_b a^2 t} \tag{4}$$

$$f_v = \frac{V}{k_v a t} \tag{5}$$

In equation (5) V is the transverse shear in the plane of the major axis of the ellipse and k_v a nondimensional coefficient that varies with the eccentricity of the ellipse. (See fig. 7.)

In figure 9 lines a and b represent the upper and lower limits of the strength in pure bending. These limiting values show the dispersion of the results of the pure bending tests and were obtained for elliptic cylinders of the average $\frac{a}{t}$ ratio in each group by interpolation of the results plotted in figure 8. At large values of $\frac{M}{r'V}$ the bending-stress diagrams approach lines a and b as an upper limit but with a tendency to plot just below them in some cases. (See fig. 9a, in particular.) The tendency for the bending-stress diagrams to plot below lines a and b seems to be more pronounced for the 0.8 than for the 0.6 ellipse although the test data are not sufficiently extensive to draw a definite conclusion. For the 0.8 ellipse in pure bending the elements on the extreme fiber and for a considerable distance toward the neutral axis tend to reach a condition of incipient failure at about the same time; whereas the 0.6 ellipse seems to have the capacity of supporting an increased load after first wrinkling. (See fig. 6.) It is therefore probable that the reduced bending strength in combined transverse shear and bending is caused by the presence of shear which might conceivably affect the strength of the 0.8 ellipse more than the strength of the 0.6 ellipse. In any case, it appears that the lower limit of the strength in pure bending is a good value to use for the bending strength of thin-walled elliptic cylinders in combined transverse shear and bending at large values of $\frac{M}{r'V}$.

At small values of $\frac{M}{r'V}$ failure occurred by shear. For comparison of the strength in transverse shear with the strength in torsion, lines c and d have been drawn in figure 9 representing the probable upper and lower limits for shear failure. These lines were drawn by plotting the equation

$$\frac{f_b}{E} = \frac{S}{E}\frac{M}{r'V} \tag{6}$$

Equation (6) is obtained from equation (3) by transposing terms, dividing by E, and substituting S for

f_V, where S is the average shearing stress at failure for an elliptic cylinder in torsion. The lines c and d for shear failure in figure 9 are shown for the two values of S obtained from figure 4 for the largest and smallest sheet thickness for each group of cylinders. When reading values from figure 4 a straight line was imagined to be drawn through the experimental points for the elliptic cylinders of 0.011-inch nominal sheet thickness.

Inspection of figure 9 shows that the bending-stress diagrams at the very low values of $\frac{M}{r'V}$, corresponding to shear failure, plot above lines c and d. As in the case of circular cylinders (reference 4), this fact indicates that the transverse shearing stress on the neutral axis at failure is higher than the shearing stress at failure in torsion. Consequently, in a like manner the ratio of the two stresses $\frac{f_V}{S}$ is plotted against $\frac{M}{r'V}$ in figure 10 for each of the tests. It will be noted that as $\frac{M}{r'V}$ approaches O the ratio $\frac{f_V}{S}$ approaches a value between 1.2 and 1.48. Thus, if S_V is the shearing stress on the neutral axis at failure in pure transverse shear and S is the shearing stress at failure for a cylinder of the same dimensions in torsion, S_V and S may be related by the following approximate equation

$$S_V = 1.25 S \tag{7}$$

The coefficient 1.25 is somewhat conservative but its use is recommended so that the same relationship will hold for both elliptic and circular cylinders. (See reference 4, equation (5).)

At intermediate values of $\frac{M}{r'V}$, near the intersection of lines a and b with lines c and d in figure 9, there is a transition from shear to bending failure. As the point of transition is approached, there seems to be developed an increased strength similar to that shown by elastic theory when the number of waves in the buckled pattern changes from n to (n±1). The increased strength at the transition seems to be confined to a small range of $\frac{M}{r'V}$ and is not always developed in the tests. For practical considerations, it therefore seems that the transition

from shear to bending failure can be given by the design chart for circular cylinders presented in reference 4 if $\frac{M}{rV}$ is replaced by $\frac{M}{r'V}$. With this change, the design chart for elliptic cylinders is given in figure 11. The solid curves in figure 9 were obtained from this chart using in one case the value of $\frac{S_b}{S_v}$ corresponding to lines a and c and in the other case the value corresponding to lines b and d. An inspection of the figures for the different groups indicates that, except for the increased strength previously discussed, these two curves represent quite well the limits of the experimental data in the transition from shear to bending failure.

In order to use the curves of figure 11 in design, it is necessary to know the loading condition $\frac{M}{r'V}$ and to be able to predict the values of S_b and S_v for the elliptic cylinder. If these three quantities are known, the maximum allowable moment and/or stress on the extreme fiber can be read from the chart as a percentage of that for pure bending. The strength in shear then need not be investigated because its effect has been taken into account by a reduced bending strength.

When checking the strength of any section between adjacent bulkheads, the largest value of $\frac{M}{r'V}$ in that section should be used to enter the chart of figure 11. This procedure tends toward conservatism and is certainly justified by the wide scattering of the test data.

CONCLUSIONS

1. In torsion, the shearing stress at failure for thin-walled elliptic cylinders was found to be equal to the shearing stress at failure for circumscribed circular cylinders of the same sheet thickness and length. Because buckling of the walls occurred at the ends of the minor axis prior to failure, the shearing stress calculated for the elliptic cylinder must be regarded as analogous to the modulus of rupture and so used in strength calculations.

2. For pure bending in the plane of the major axis,

the calculated stress on the extreme fiber at failure was
greater than the corresponding stress for circumscribed
circular cylinders of the same sheet thickness and length.
As in the case of circular cylinders, it was found that
slight imperfections caused the test results to scatter
widely. In view of this scattering, the bending strength
must be considered somewhat indefinite and should therefore
be estimated after consideration of all the experimental
data for both elliptic and circular cylinders. Design
values may then be chosen more or less conservatively as
desired.

3. For combined transverse shear and bending in the
plane of the major axis, the strength was dependent upon
the loading condition as described by the term $\frac{M}{r'V}$. At
small values of $\frac{M}{r'V}$, failure occurred in shear and, as
$\frac{M}{r'V}$ approached zero (a condition of pure transverse shear),
the shearing stress on the neutral axis at failure as cal-
culated by the ordinary beam theory was approximately 1.25
times the shearing stress at failure in torsion.

At large values of $\frac{M}{r'V}$, failure occurred in bend-
ing and the strength developed was approximately equal to
the lowest strengths developed for cylinders of the same
dimension in pure bending.

At intermediate values of $\frac{M}{r'V}$, there was a transi-
tion from shear to bending failure. For use in calculat-
ing the strength of elliptic cylinders in combined trans-
verse shear and bending it was found that a chart previous-
ly derived for circular cylinders may be used.

Langley Memorial Aeronautical Laboratory,
      National Advisory Committee for Aeronautics,
            Langley Field, Va., April 2, 1935.

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TABLE I

RESULTS OF TORSION TESTS ON ELLIPTIC CYLINDERS

For all cylinders, Poisson's ratio $\mu = 0.3$

$$\frac{b}{a} = 0.8$$

Specimen No.	t	l	T	S	$(1-\mu^2)\frac{S}{E}\frac{l^2}{t^2}$	$\sqrt{1-\mu^2}\frac{l^2}{td}$	Remarks
	in.	in.	lb.-in.	lb./sq.in.			
279	0.0104	3.75	5510	1875	21.4	86.0	First wrinkle
			7790	2650	30.2		Failure
281	.0117	7.50	5710	1730	62.2	306.0	First wrinkle
			6150	1860	67.0		Failure
283	.0115	11.25	3900	1200	100.5	700.0	First wrinkle
			4860	1496	125.3		Failure
285	.0113	15.00	2220	694	107.1	1274.0	First wrinkle
			4000	1250	193.0		Failure
287	.0116	22.50	2100	640	211.0	2780.0	First wrinkle
			3720	1133	373.0		Failure
309	.0158	3.75	15300	3425	16.9	56.6	First wrinkle
			19260	4310	21.3		Failure
311	.0162	7.50	9660	2110	39.5	221.0	First wrinkle
			13980	3050	57.2		Failure
313	.0162	11.25	9512	2080	87.8	497.0	First wrinkle
			11760	2570	108.5		Failure
315	.0161	15.00	3920	856	65.0	890.0	First wrinkle
			10440	2295	174.4		Failure
317	.0160	22.50	7120	1575	272.0	2010.0	First wrinkle
			8180	1810	313.0		Failure
349	.0241	3.75	43488	6380	13.5	37.1	First wrinkle
			48488	7110	15.1		Failure
351	.0246	7.50	29488	4240	34.5	145.5	First wrinkle
			32288	4640	37.8		Failure
353	.0237	11.25	20408	3050	60.2	339.5	First wrinkle
			23088	3440	67.8		Failure

TABLE I (Cont.)

$$\frac{b}{a} = 0.8$$

Specimen No.	t	l	T	S	$(1-\mu^2)\frac{S}{E}\frac{l^2}{t^2}$	$\sqrt{1-\mu^2}\ \frac{l^2}{td}$	Remarks
	in.	in.	lb.-in.	lb./sq.in.			
355	0.0247	15.00	14408	2060	66.5	580.0	First wrinkle
			22508	3225	104.0		Failure
357	.0240	22.50	17868	2635	203.0	1342.0	First wrinkle
			18588	2740	210.0		Failure

$$\frac{b}{a} = 0.6$$

Specimen No.	t	l	T	S	$(1-\mu^2)\frac{S}{E}\frac{l^2}{t^2}$	$\sqrt{1-\mu^2}\ \frac{l^2}{td}$	Remarks
280	0.0104	3.75	3310	1502	17.1	86.0	First wrinkle
			6790	3080	35.1		Failure
282	.0114	7.50	1900	787	29.8	314.0	First wrinkle
			5420	2240	84.9		Failure
284	.0115	11.25	1860	763	63.9	700.0	First wrinkle
			3740	1533	128.4		Failure
286	.0114	15.00	2100	870	131.6	1263.0	First wrinkle
			3200	1325	200.5		Failure
288	.0116	22.50	2140	871	287.0	2780.0	First wrinkle
			2880	1170	385.0		Failure
310	.0157	3.75	9260	2785	13.9	57.0	First wrinkle
			16700	5010	25.0		Failure
312	.0167	7.50	7660	2165	38.2	214.0	First wrinkle
			11960	3375	59.6		Failure
314	.0159	11.25	5120	1520	66.6	506.0	First wrinkle
			8480	2520	110.3		Failure
316	.0161	15.00	6320	1853	140.7	890.0	First wrinkle
			7720	2265	172.0		Failure
318	.0161	22.50	5720	1678	287.0	2000.0	First wrinkle
			6520	1910	327.0		Failure
350	.0238	3.75	27488	5445	11.8	37.6	First wrinkle
			35328	7000	15.2		Failure

TABLE I (Cont.)

$$\frac{b}{a} = 0.6$$

Specimen No.	t	l	T	S	$(1-\mu^2)\frac{S}{E}\frac{l^2}{t^2}$	$\sqrt{1-\mu^2}\,\frac{l^2}{td}$	Remarks
	in.	in.	lb.-in.	lb./sq.in.			
352	0.0235	7.50	20488	4110	36.7	152.5	First wrinkle
			25048	5030	44.9		Failure
354	.0237	11.25	12407	2470	48.7	339.5	First wrinkle
			18008	3580	70.6		Failure
356	.0243	15.00	12408	2410	80.4	590.0	First wrinkle
			17128	3325	111.0		Failure
358	.0235	22.50	10868	2180	175.0	1370.0	First wrinkle
			13708	2745	220.0		Failure

TABLE II

RESULTS OF PURE BENDING TESTS ON ELLIPTIC CYLINDERS SHOWING CORRECTIONS
MADE FOR THE EFFECT OF A SMALL TENSION P ON THE SPECIMEN

$$\frac{b}{a} = 0.8$$

Spec. No.	t	$\frac{a}{t}$	l	P	M	Stress caused by tension P	f_b	S_b	$\frac{S_b}{E}$	Remarks
	in.		in.							
290	0.0110	682	3.75	93.0	11335	198	6820	6622	0.000637	Failure
291	.0113	664	7.50	83.5	13305	173	7790	7617	.000732	Failure
293	.0113	664	11.25	83.5	10945	173	6410	6237	.000600	First wrinkle
					11185		6550	6377	.000613	Failure
295	.0110	682	15.00	76.0	6935	162	4172	4010	.000386	First wrinkle
					10635		6400	6238	.000600	Failure
297	.0110	682	22.50	74.0	8535	158	5135	4977	.000479	First wrinkle
					11055		6650	6492	.000624	Failure
307	.0166	452	3.75	80.0	28855	113	11510	11397	.001096	Failure
305	.0161	466	7.50	80.0	22355	116	9185	9069	.000872	Failure
303	.0164	457	11.25	80.0	23175	114	9360	9246	.000889	First wrinkle
					27055		10930	10816	.001040	Failure
301	.0167	449	15.00	74.0	27655	104	10970	10866	.001045	First wrinkle
					28915		11470	11366	.001093	Failure
299	.0168	446	22.50	74.0	24175	103	9530	9427	.000907	Failure
339	.0245	306	3.75	95.0	59435	91	16065	15974	.001536	Failure
341	.0252	297	7.50	95.0	62195	88	16335	16247	.001562	Failure
343	.0235	319	11.25	82.0	56355	82	15880	15798	.001520	Failure
345	.0248	302	15.00	74.0	59875	70	15985	15915	.001531	Failure
347	.0234	320	22.50	74.0	52315	74	14810	14736	.001417	Failure

$$\frac{b}{a} = 0.6$$

Spec. No.	t	$\frac{a}{t}$	l	P	M	Stress caused by tension P	f_b	S_b	$\frac{S_b}{E}$	Remarks
289	0.0110	682	3.75	87.0	9595	207	6910	6703	0.000644	First wrinkle
					11255		8110	7903	.000760	Failure
292	.0107	701	7.50	77.5	9965	190	7385	7195	.000692	First wrinkle
					11165		8270	8080	.000777	Failure
294	.0113	664	11.25	77.5	9645	180	6765	6585	.000633	First wrinkle
					13025		9140	8960	.000861	Failure
296	.0110	682	15.00	70.0	9135	166	6585	6419	.000617	First wrinkle
					11955		8620	8454	.000813	Failure
298	.0110	682	22.50	68.0	11355	162	8180	8018	.000771	Failure
308	.0168	446	3.75	74.0	29775	115	14050	13935	.001340	First wrinkle
					31935		15085	14970	.001439	Failure
306	.0163	460	7.50	74.0	23415	119	11400	11281	.001085	First wrinkle
					26395		12855	12736	.001224	Failure

TABLE II (Cont.)

$$\frac{b}{a} = 0.6$$

Spec. No.	t	$\frac{a}{t}$	l	P	M	Stress caused by tension P	f_b	S_b	$\frac{S_b}{E}$	Remarks
	in.		in.							
304	0.0167	449	11.25	74.0	28995	116	13790	13674	0.001315	Failure
302	.0168	446	15.00	68.0	27615	105	13040	12935	.001244	Failure
300	.0169	444	22.50	68.0	24675	105	11590	11485	.001104	First wrinkle
					28015		13160	13055	.001255	Failure
340	.0240	312	3.75	89.0	57015	97	18850	18753	.001803	Failure
342	.0248	302	7.50	89.0	61075	94	19550	19456	.001871	Failure
344	.0234	320	11.25	76.0	51415	85	17450	17365	.001670	First wrinkle
					51815		17580	17495	.001682	Failure
346	.0245	306	15.00	68.0	58995	73	19110	19037	.001831	Failure
348	.0236	318	22.50	68.0	45075	75	15150	15075	.001449	First wrinkle
					49275		16565	16490	.001586	Failure

TABLE III

RESULTS OF COMBINED TRANSVERSE SHEAR AND BENDING TESTS ON ELLIPTIC CYLINDERS

Group 1 $\frac{b}{a}$ = 0.8   r' = 6.48 $\frac{l}{a}$ = 0.50

Spec. No.	t	$\frac{a}{t}$	V	M	f_v	f_b	$\frac{M}{r'V}$	$\frac{f_b}{E}$	Remarks
	in.		lb.	lb.-in.	lb./sq.in.	lb./sq.in.			
379	0.0119	630	569	2558	2072	1423	0.69	0.000137	First wrinkle
			1122	5323	4085	2962	.73	.000285	Failure
381	.0113	664	459	4288	1756	2510	1.44	.000241	First wrinkle
			807	7420	3090	4345	1.42	.000418	Failure
383	.0113	664	437	6283	1672	3680	2.22	.000354	First wrinkle
			719	9943	2752	5822	2.12	.000560	Failure
385	.0119	630	452	9283	1645	5170	3.17	.000497	First wrinkle
			680	13403	2475	7460	3.03	.000718	Failure
387	.0111	676	244	7320	953	4370	4.63	.000420	First wrinkle
			415	11260	1619	6720	4.16	.000646	Failure
389	.0111	676	175	7158	683	4272	6.31	.000411	First wrinkle
			307	10858	1198	6480	5.45	.000623	Failure
391	.0115	652	197	8723	742	5025	6.83	.000483	First wrinkle
			253	10563	953	6080	6.42	.000585	Failure
393	.0117	641	83	5954	307	3370	11.07	.000324	First wrinkle
			207	10664	766	6040	7.94	.000581	Failure
395	.0110	682	139	9423	547	5670	10.47	.000545	First wrinkle
			161	10353	634	6230	9.87	.000599	Failure
397	.0110	682	123	9971	484	6000	12.42	.000578	Failure

TABLE III (Cont.)

Group 2 $\dfrac{b}{a}$ = 0.6 r' = 5.60 $\dfrac{l}{a}$ = 0.50

Spec. No.	t	$\dfrac{a}{t}$	V	M	f_v	f_b	$\dfrac{M}{r'V}$	$\dfrac{f_b}{E}$	Remarks
			lb.	lb.-in.	lb./sq.in.	lb./sq.in.			
380	0.0120	625	463	2027	1710	1341	0.78	0.000129	First wrinkle
			1219	5807	4505	3840	.86	.000370	Failure
382	.0114	658	513	4749	1994	3305	1.65	.000318	First wrinkle
			869	7953	3380	5535	1.64	.000532	Failure
384	.0114	658	480	6790	1866	4727	2.52	.000455	First wrinkle
			815	11152	3170	7765	2.46	.000747	Failure
386	.0119	630	293	6342	1092	4230	3.87	.000407	First wrinkle
			707	13812	2636	9210	3.51	.000886	Failure
388	.0112	670	343	9484	1357	6725	4.94	.000647	First wrinkle
			381	10359	1507	7340	4.89	.000706	Failure
390	.0111	676	313	10887	1250	7785	6.21	.000749	First wrinkle
			413	13687	1650	9790	5.95	.000941	Failure
392	.0115	652	113	5782	436	4000	9.14	.000385	First wrinkle
			330	12942	1272	8935	7.04	.000858	Failure
394	.0115	652	247	11983	953	8280	8.66	.000796	First wrinkle
			275	13053	1060	9010	8.53	.000866	Failure
396	.0109	688	155	9882	630	7180	11.38	.000690	First wrinkle
			205	12022	834	8755	10.52	.000842	Failure
398	.0111	676	133	10200	532	7300	13.70	.000702	First wrinkle
			173	12100	692	8650	12.55	.000832	Failure

TABLE III (Cont.)

Group 3 $\frac{b}{a}$ = 0.8   r' = 6.48 $\frac{l}{a}$ = 1.0

Spec. No.	t	$\frac{a}{t}$	V	M	f_v	f_b	$\frac{M}{r'V}$	$\frac{f_b}{E}$	Remarks
	in.		lb.	lb.-in.	lb./sq.in.	lb./sq.in.			
319	0.0106	707	271	2004	1106	1252	1.14	0.000120	First wrinkle
			551	4384	2250	2738	1.22	.000264	Failure
321	.0113	664	491	5670	1880	3323	1.78	.000319	First wrinkle
			647	7454	2478	4367	1.76	.000420	Failure
323	.0106	707	258	4284	1054	2677	2.56	.000257	First wrinkle
			537	8474	2192	5295	2.42	.000509	Failure
325	.0106	707	279	6162	1139	3850	3.41	.000370	First wrinkle
			420	8843	1715	5525	3.22	.000532	Failure
327	.0112	670	403	10570	1557	6250	4.05	.000601	First wrinkle
			413	10806	1596	6420	4.00	.000614	Failure
329	.0106	707	195	6759	796	4220	5.35	.000406	First wrinkle
			268	8728	1094	5453	5.00	.000525	Failure
331	.0114	658	145	6437	551	3738	6.86	.000359	First wrinkle
			271	10337	1029	6000	5.85	.000578	Failure
333	.0107	701	211	9765	854	6045	7.14	.000582	First wrinkle
			215	9905	871	6130	7.07	.000590	Failure
335	.0114	658	213	11521	810	6700	8.34	.000644	First wrinkle
			217	11685	825	6785	8.26	.000653	Failure
337	.0107	701	77	7490	312	4636	15.01	.000446	First wrinkle
			83	7780	336	4815	14.40	.000463	Failure

TABLE III (Cont.)

Group 4 $\dfrac{b}{a} = 0.6$ $r' = 5.60$ $\dfrac{l}{a} = 1.0$

Spec. No.	t	$\dfrac{a}{t}$	V	M	f_v	f_b	$\dfrac{M}{r'V}$	$\dfrac{f_b}{E}$	Remarks
	in.		lb.	lb.-in.	lb./sq.in.	lb./sq.in.			
320	0.0113	664	367	2820	1439	1979	1.37	0.000190	First wrinkle
			700	5655	2746	3970	1.45	.000382	Failure
322	.0106	707	307	3535	1283	2647	2.06	.000255	First wrinkle
			595	6855	2488	5130	2.07	.000493	Failure
324	.0104	721	287	4680	1223	3570	2.91	.000343	First wrinkle
			537	8435	2288	6435	2.81	.000619	Failure
326	.0105	714	221	4993	932	3770	4.03	.000363	First wrinkle
			417	8723	1759	6590	3.75	.000634	Failure
328	.0113	664	292	7931	1145	5565	4.85	.000535	First wrinkle
			512	12996	2008	9125	4.55	.000878	Failure
330	.0106	707	327	10210	1367	7650	5.57	.000736	First wrinkle
			381	11668	1593	8740	5.49	.000840	Failure
332	.0114	658	297	11008	1154	7660	6.61	.000737	First wrinkle
			388	13829	1508	9630	6.39	.000926	Failure
334	.0105	714	203	9336	856	7050	8.21	.000678	First wrinkle
			241	10656	1016	8050	7.94	.000775	Failure
336	.0114	658	219	11572	852	8060	9.42	.000775	First wrinkle
			242	12515	941	8710	9.28	.000838	Failure
338	.0105	714	91	7931	384	5990	15.55	.000576	First wrinkle
			122	9448	515	7180	13.90	.000687	Failure

TABLE III (Cont.)

Group 5 $\frac{b}{a}$ = 0.8   r' = 6.48 $\frac{l}{a}$ = 2.0

Spec. No.	t	$\frac{a}{t}$	V	M	f_v	f_b	$\frac{M}{r'V}$	$\frac{f_b}{E}$	Remarks
	in.		lb.	lb.-in.	lb./sq.in.	lb./sq.in.			
359	0.0119	630	437	6137	1590	3415	2.17	0.000328	First wrinkle
			527	7487	1919	4168	2.18	.000401	Failure
361	.0116	647	502	2514	1873	1434	.77	.000138	First wrinkle
			521	2652	1945	1513	.78	.000146	Failure
363	.0110	682	270	2487	1062	1496	1.42	.000144	First wrinkle
			443	4562	1745	2744	1.58	.000264	Failure
365	.0110	682	251	4424	987	2662	2.72	.000256	First wrinkle
			409	7275	1610	4380	2.73	.000422	Failure
367	.0118	636	407	9822	1493	5510	3.72	.000530	First wrinkle
			423	10222	1552	5740	3.73	.000552	Failure
369	.0110	682	313	9767	1232	5880	4.79	.000566	Failure
371	.0114	658	307	11701	1166	6800	5.88	.000654	First wrinkle
			315	11941	1196	6935	5.83	.000667	Failure
373	.0113	664	269	12351	1029	7235	7.05	.000696	Failure
375	.0111	676	140	8582	546	5120	9.46	.000492	First wrinkle
			161	9482	629	5660	9.05	.000544	Failure
377	.0114	658	192	11914	729	6920	9.58	.000665	First wrinkle
			194	12014	737	6980	9.50	.000671	Failure

TABLE III (Cont.)

Group 6 $\frac{b}{a}$ = 0.6   r' = 5.60 $\frac{l}{a}$ = 2.0

Spec. No.	t	$\frac{a}{t}$	V	M	f_v	f_b	$\frac{M}{r'V}$	$\frac{f_b}{E}$	Remarks
	in.		lb.	lb.-in.	lb./sq.in.	lb./sq.in.			
360	0.0119	630	383	5332	1428	3555	2.49	0.000342	First wrinkle
			498	7052	1857	4700	2.54	.000452	Failure
362	.0114	658	323	1222	1256	851	.68	.000082	First wrinkle
			501	2557	1949	1780	.92	.000171	Failure
364	.0111	676	213	1827	851	1306	1.53	.000126	First wrinkle
			433	3867	1730	2764	1.60	.000266	Failure
366	.0111	676	291	5134	1162	3670	3.15	.000353	First wrinkle
			424	7534	1694	5388	3.19	.000518	Failure
368	.0119	630	343	8307	1278	5540	4.32	.000533	First wrinkle
			447	10727	1666	7155	4.32	.000688	Failure
370	.0110	682	181	6012	729	4333	5.93	.000416	First wrinkle
			348	10662	1402	7690	5.49	.000740	Failure
372	.0116	647	217	8626	830	5910	7.10	.000568	First wrinkle
			283	10456	1082	7155	6.64	.000688	Failure
374	.0114	658	222	10446	862	7270	8.39	.000700	First wrinkle
			307	13686	1194	9530	8.00	.000916	Failure
376	.0111	676	215	11617	858	8310	9.65	.000800	First wrinkle
			245	12877	979	9210	9.45	.000885	Failure
378	.0114	658	217	12939	844	9010	10.70	.000866	Failure
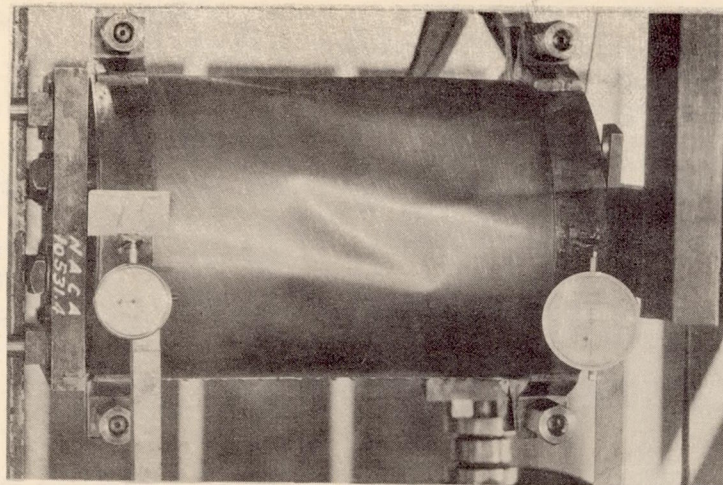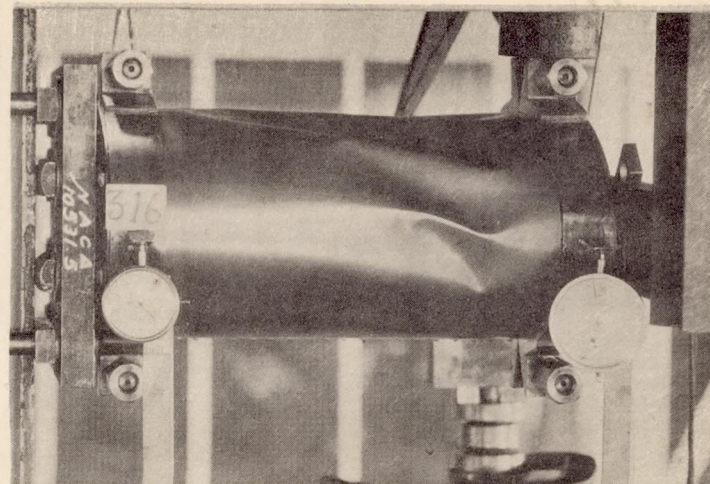
$\frac{b}{a} = 0.8$

$\frac{b}{a} = 0.6$

Figure 1.-Elliptic cylinders after failure in torsion.

$\frac{b}{a} = 0.8$
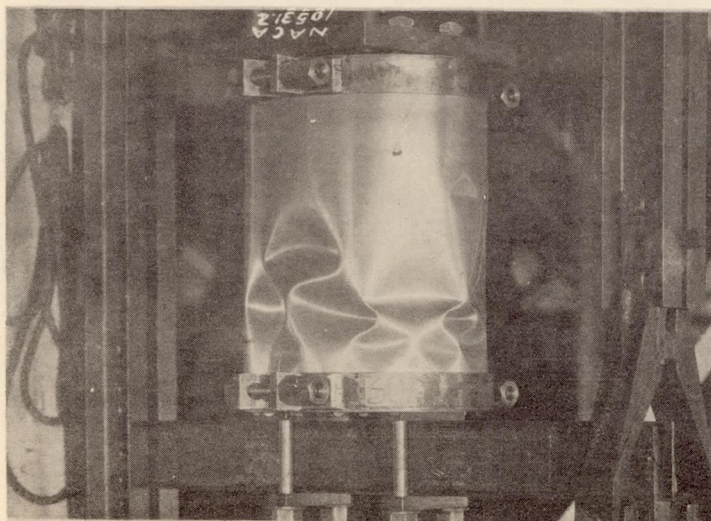
$\frac{b}{a} = 0$
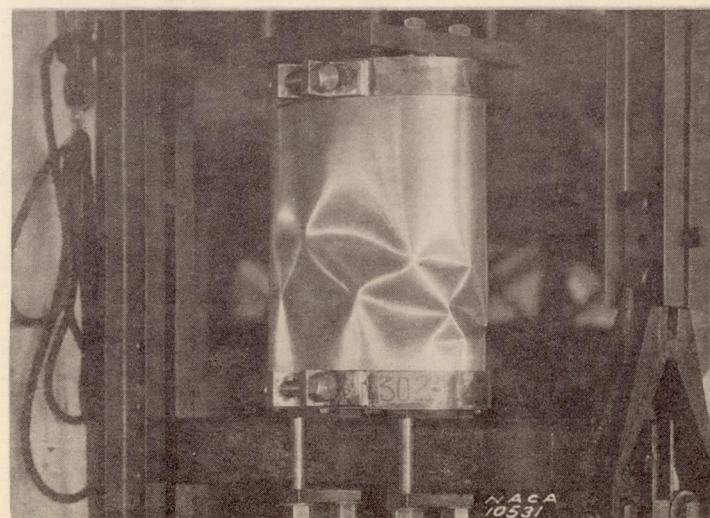
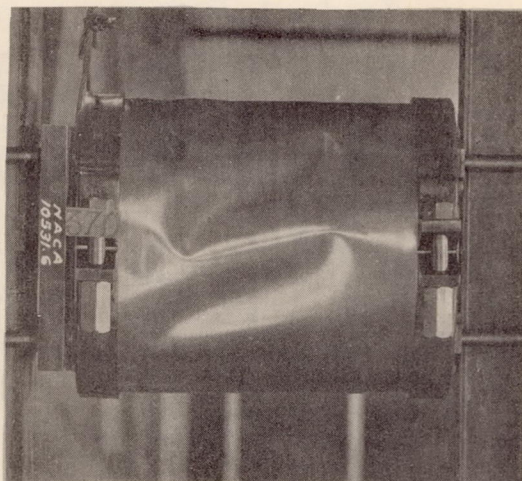Figure 2.-Elliptic cylinders after failure in pure bending.

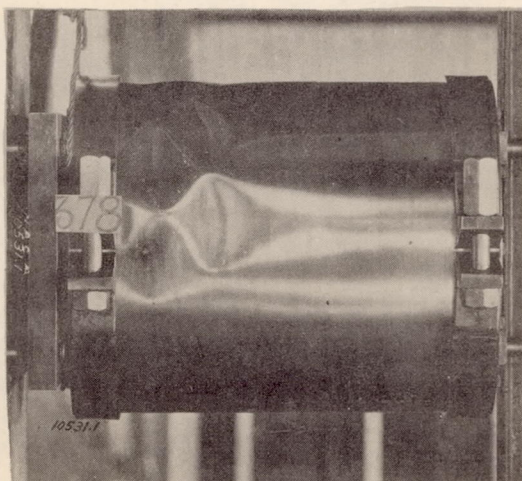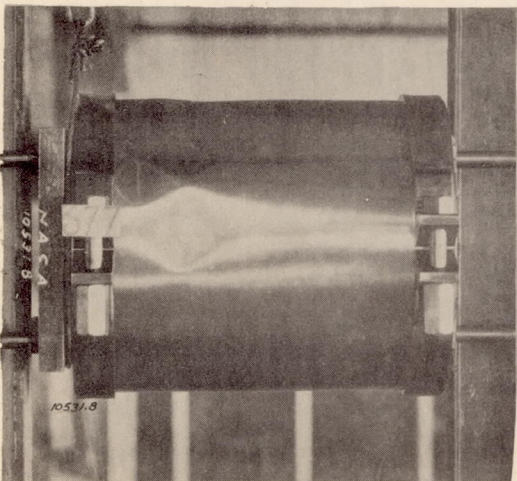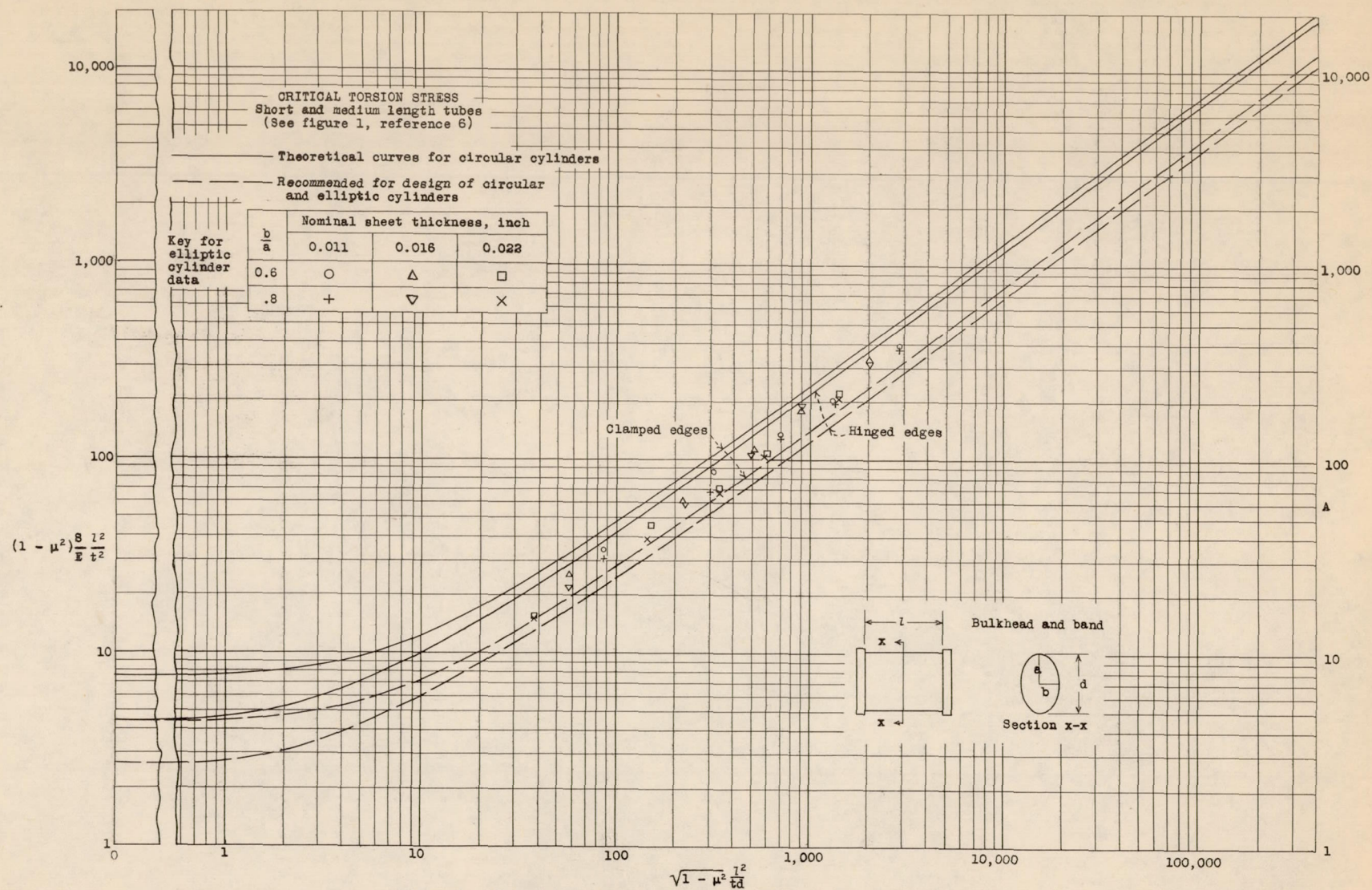$$\frac{b}{a} = 0.8 \qquad\qquad\qquad\qquad \frac{b}{a} = 0.6$$
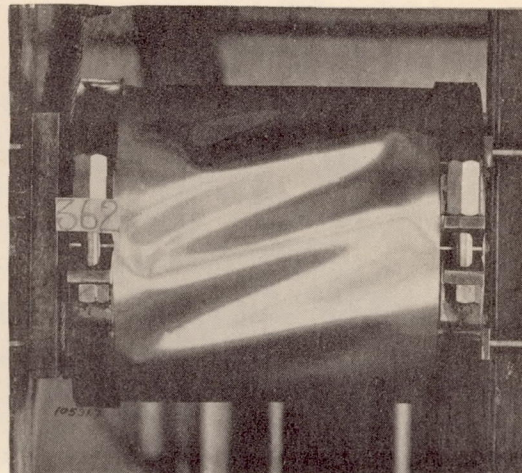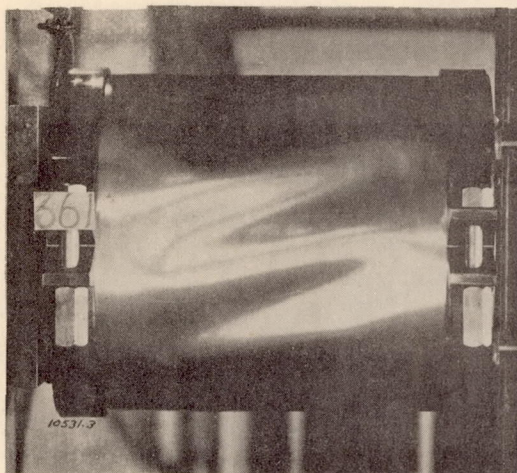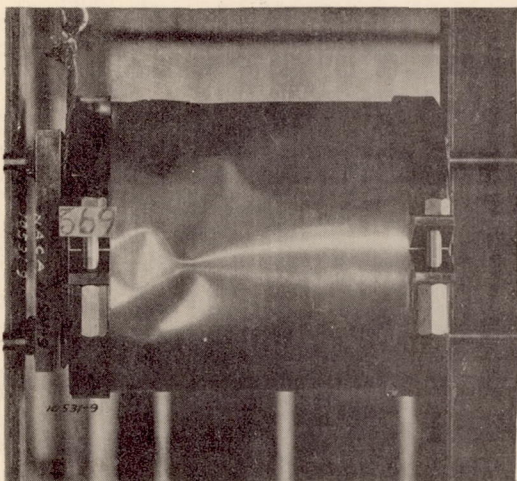
Figure 3.-Side view of elliptic cylinders after failure in combined
transverse shear and bending. (Photographs show the tran-
sition from shear to bending failure as M/r'V varies from small to
large values).

Fig. 4

CRITICAL TORSION STRESS
Short and medium length tubes
(See figure 1, reference 6)

———————— Theoretical curves for circular cylinders

———————— Recommended for design of circular
and elliptic cylinders

Key for elliptic cylinder data	Nominal sheet thickness, inch		
$\frac{b}{a}$	0.011	0.016	0.022
0.6	O	△	□
.8	+	▽	×

Clamped edges

Hinged edges

$(1 - \mu^2)\dfrac{S}{E}\dfrac{l^2}{t^2}$

$\sqrt{1 - \mu^2}\,\dfrac{l^2}{td}$

Bulkhead and band

Section x-x

Figure 4. – Shearing stress at failure S for elliptic cylinders in torsion.
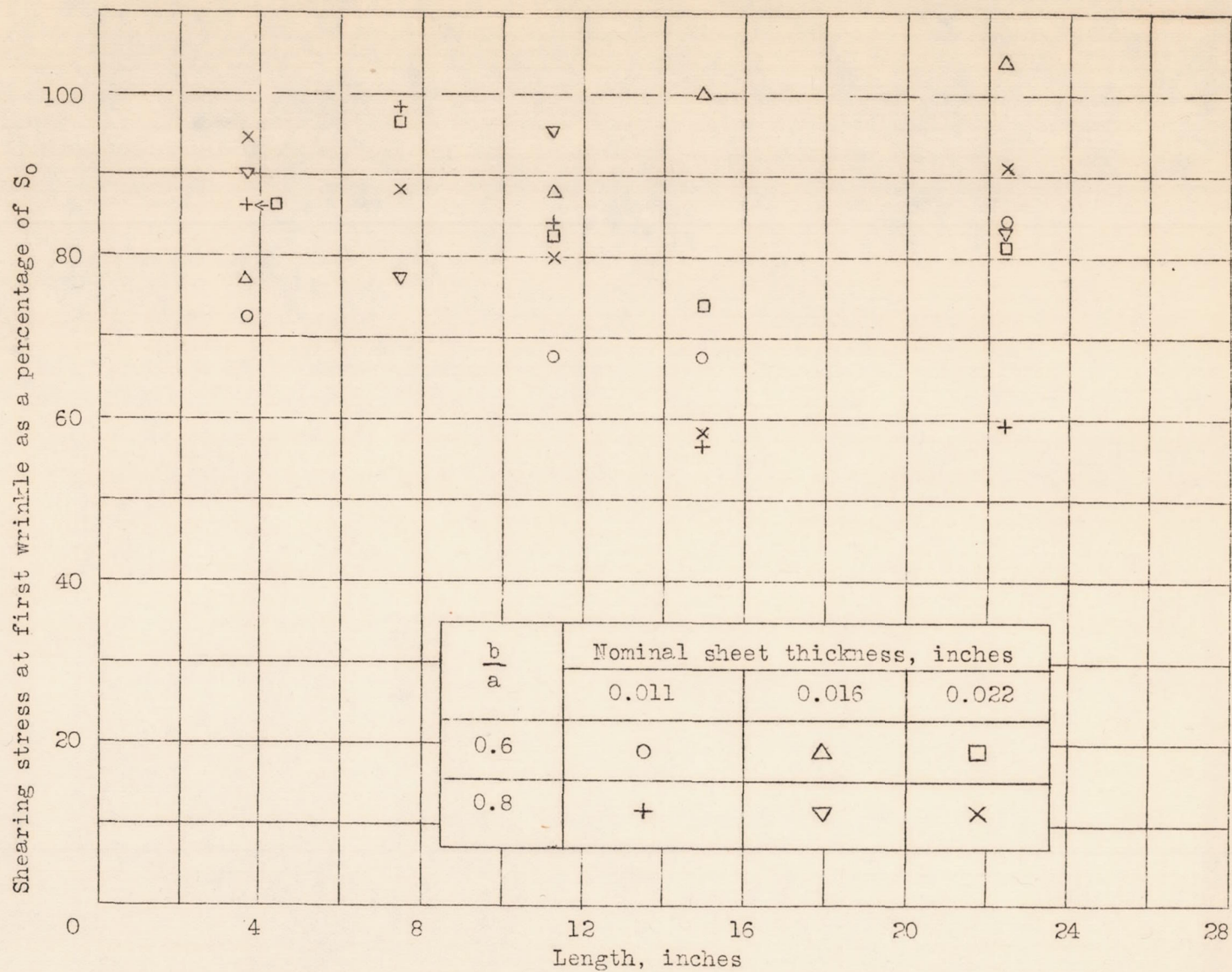
Figure 5.- Shearing stress at first wrinkle for elliptic cylinders in torsion.

Fig. 6

N.A.C.A. Technical Note No. 527

$\frac{b}{a} = 0.8$

$\frac{b}{a} = 0.6$

Distance from neutral axis as a percentage of a.

Radius of curvature

Radius of curvature

$\frac{a}{\text{Radius of curvature}}$
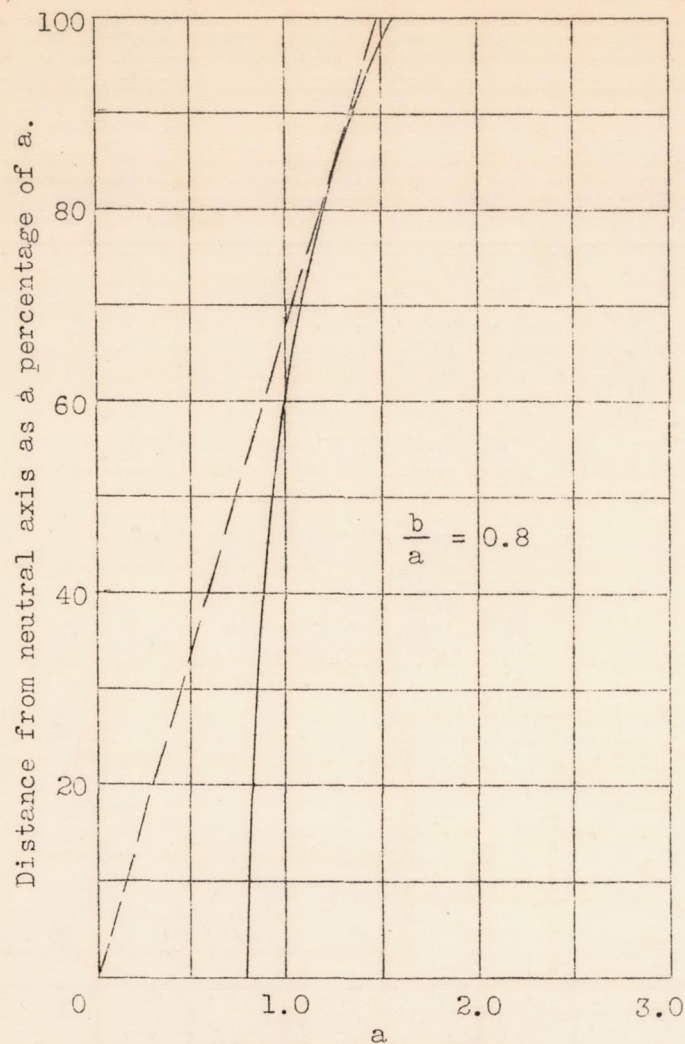
— — — — Stress distribution at first wrinkle by ordinary theory of bending.

————— Critical stress proportional to curvature.
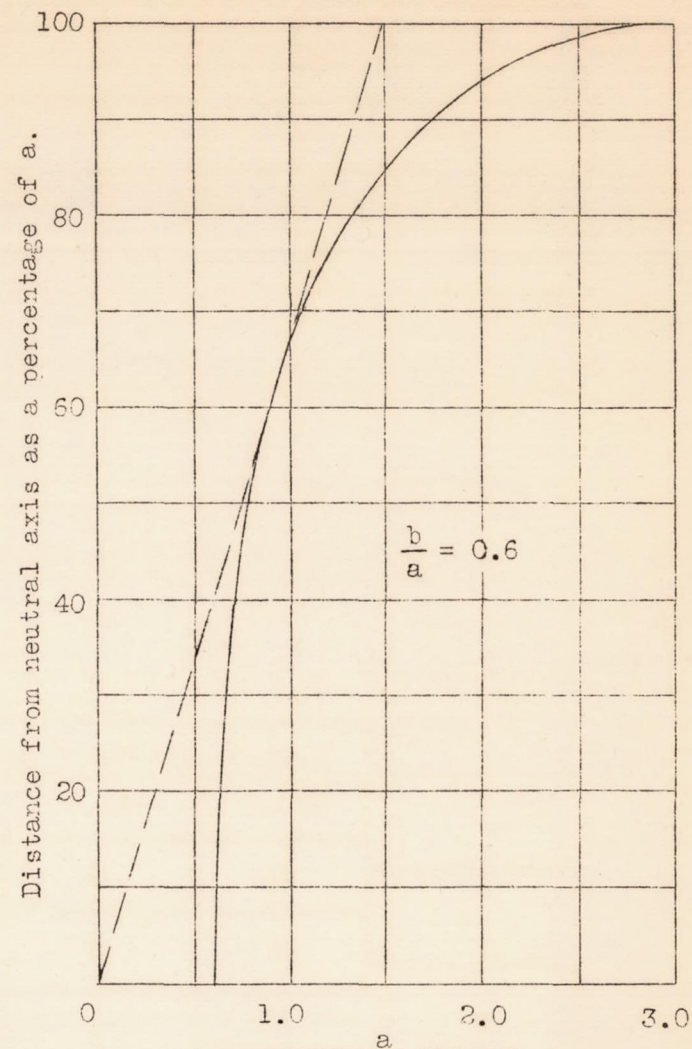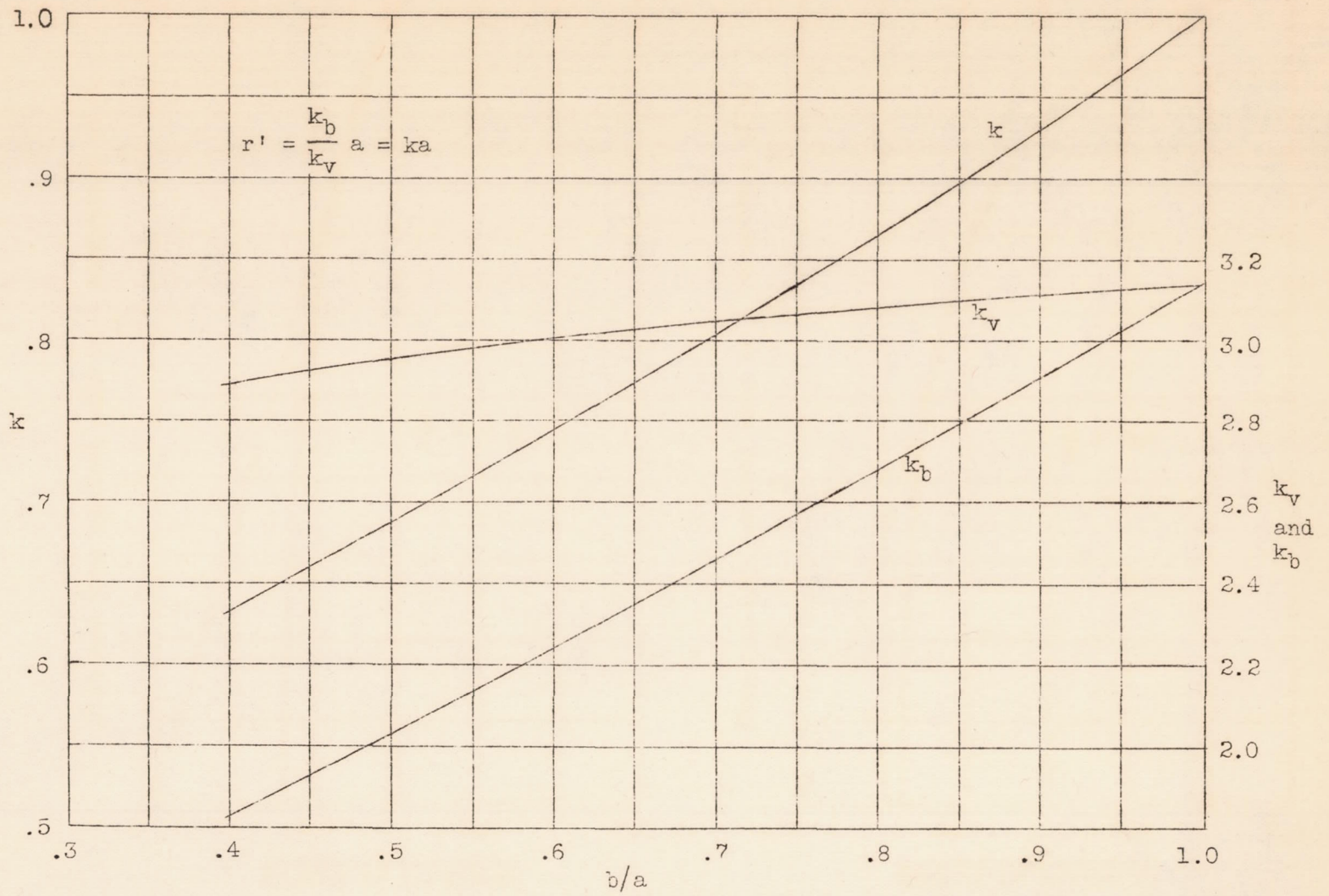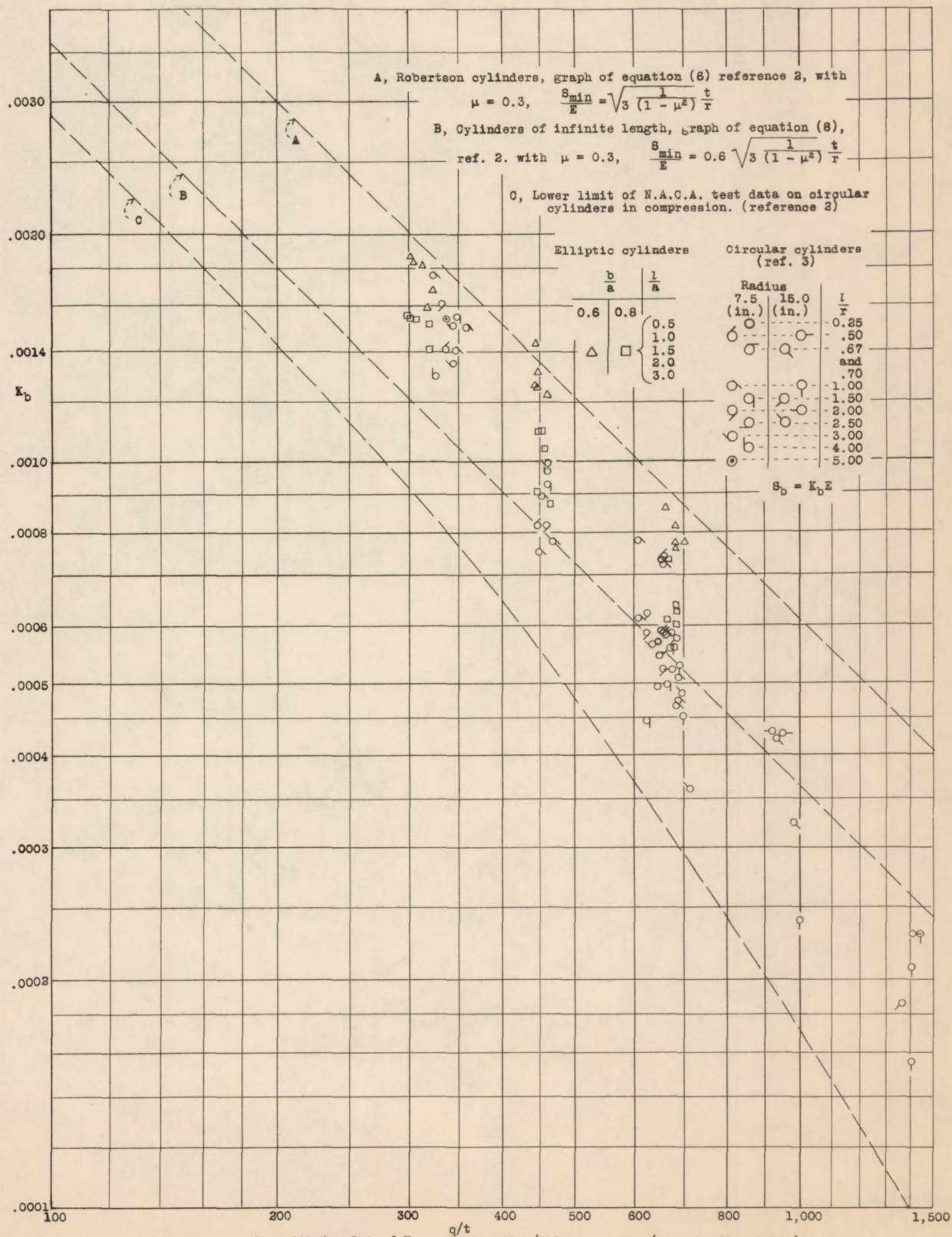
Figure 6.- Stress distribution in cylinders.

Fig. 7

$$r' = \frac{k_b}{k_v} \, a = ka$$

Figure 7.- Stress coefficients.

A, Robertson cylinders, graph of equation (6) reference 2, with

$\mu = 0.3, \quad \dfrac{S_{min}}{E} = \sqrt{3\ \dfrac{1}{(1 - \mu^2)}}\ \dfrac{t}{r}$

B, Cylinders of infinite length, graph of equation (8),

ref. 2. with $\mu = 0.3, \quad \dfrac{S_{min}}{E} = 0.6 \sqrt{3\ \dfrac{1}{(1 - \mu^2)}}\ \dfrac{t}{r}$

C, Lower limit of N.A.C.A. test data on circular cylinders in compression. (reference 2)

Elliptic cylinders		Circular cylinders (ref. 3)		
$\dfrac{b}{a}$	$\dfrac{l}{a}$	Radius 7.5 (in.)	15.0 (in.)	$\dfrac{l}{r}$
0.6	0.8			
				0.25
				.50
				.67 and .70
				1.00
				1.50
				2.00
				2.50
				3.00
				4.00
				5.00

0.5
1.0
1.5
2.0
3.0

$S_b = K_b E$

K_b

.0030
.0020
.0014
.0010
.0008
.0006
.0005
.0004
.0003
.0002
.0001

100   200   300   400   500   600   800   1,000   1,500

q/t

Figure 8. – Logarithmic plot of K_b against radius/thickness ratio (pure bending tests).

Figure 9a.- Bending-stress diagrams for elliptic cylinders in combined transverse shear and bending.
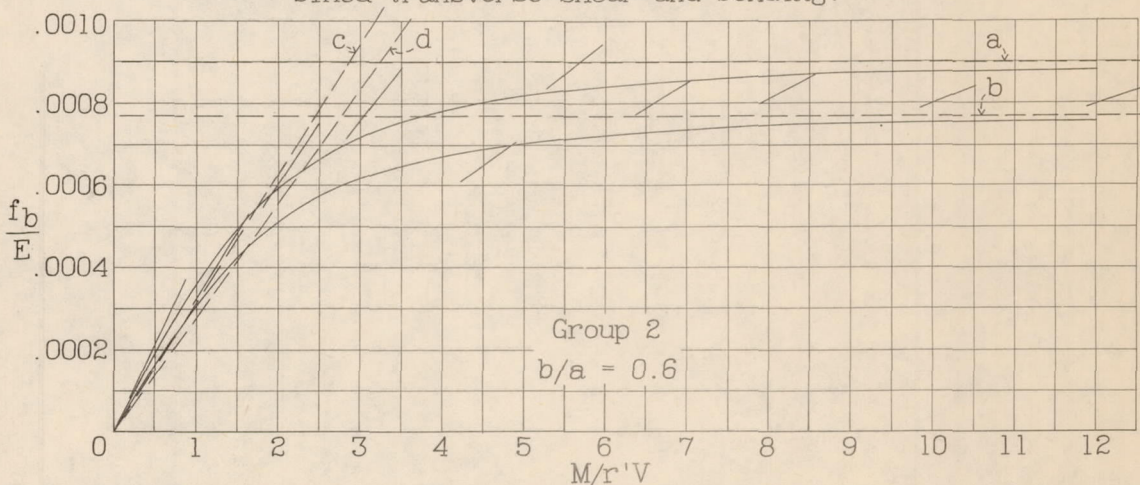
Figure 9b.- Bending-stress diagrams for elliptic cylinders in combined transverse shear and bending.
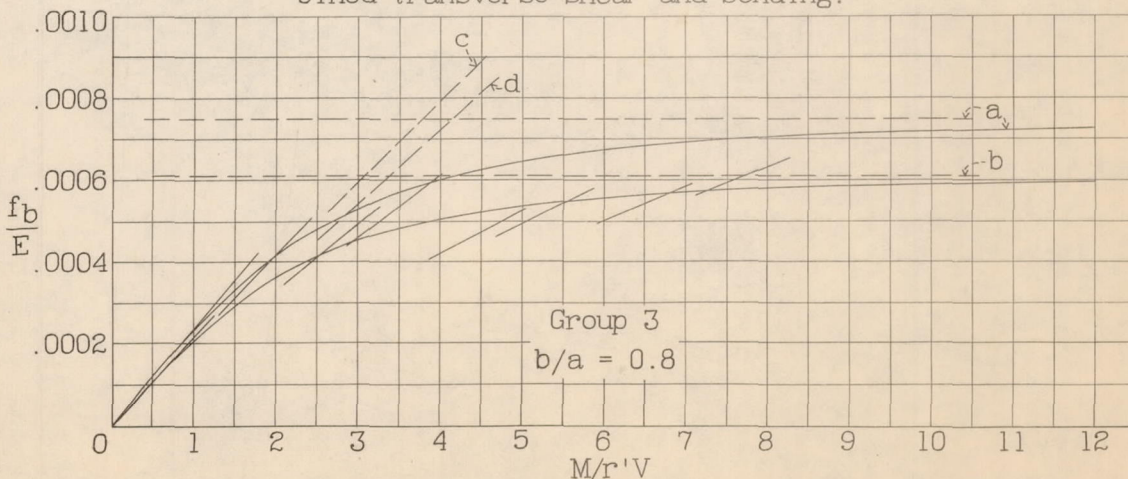
Figure 9c.- Bending-stress diagrams for elliptic cylinders in combined transverse shear and bending.
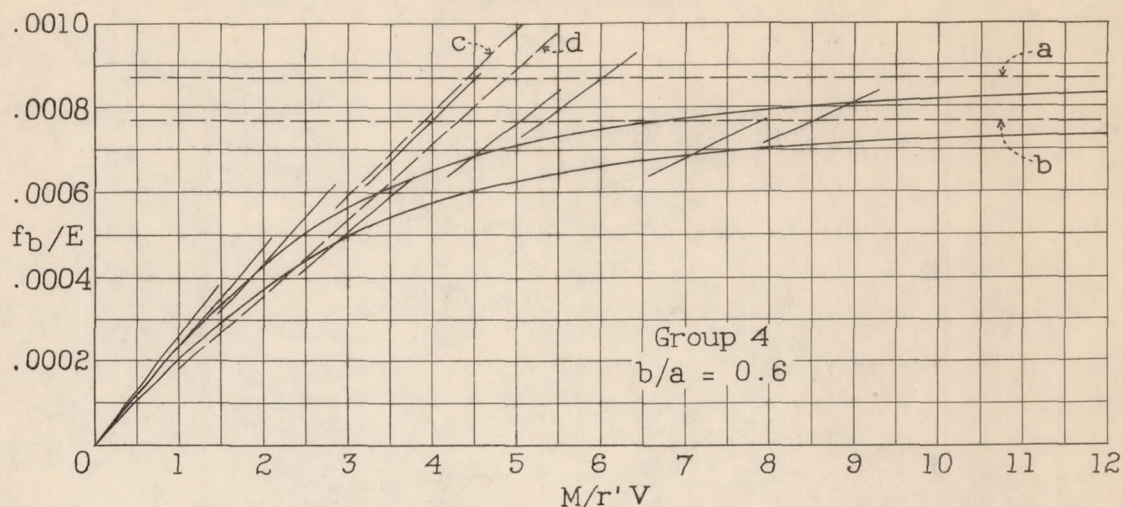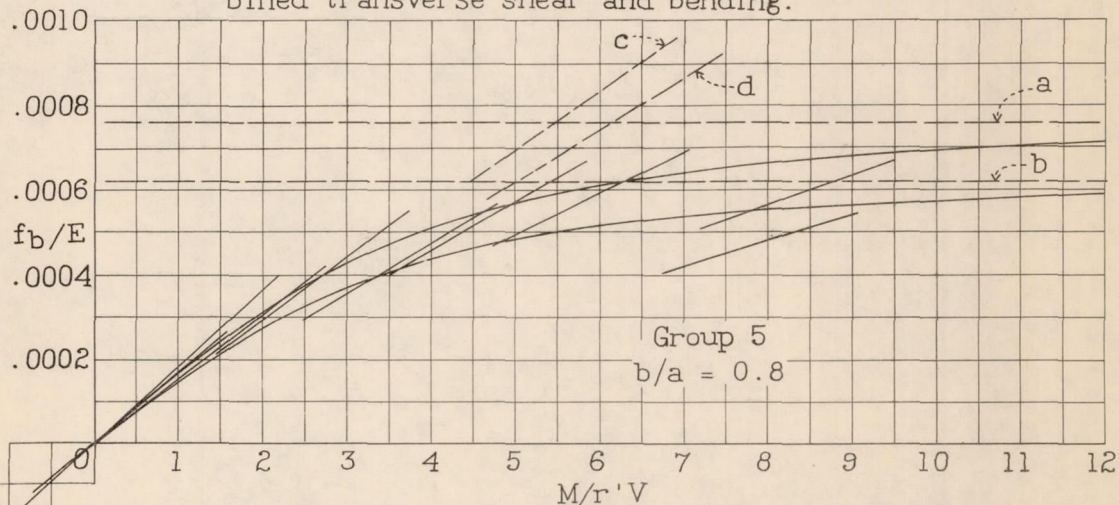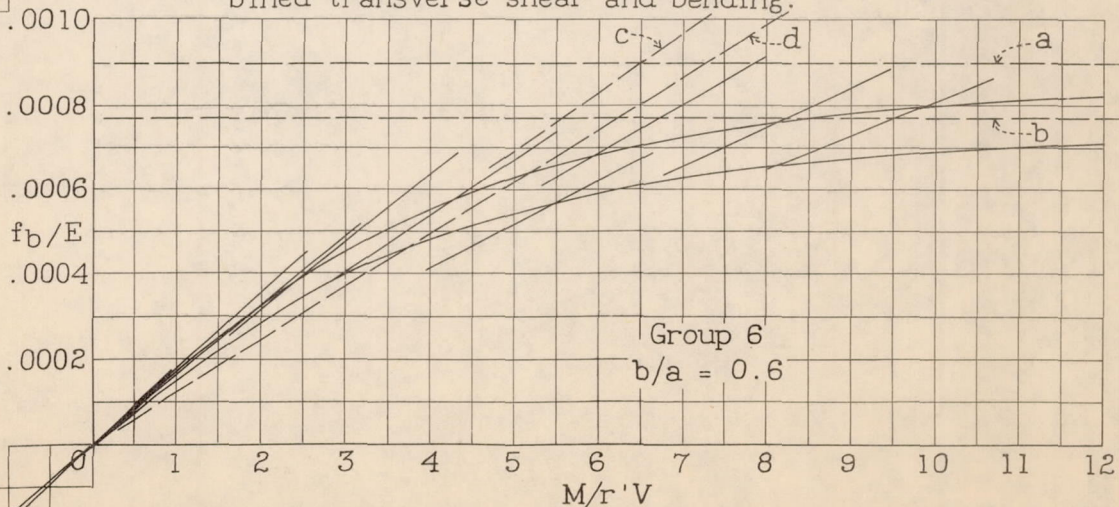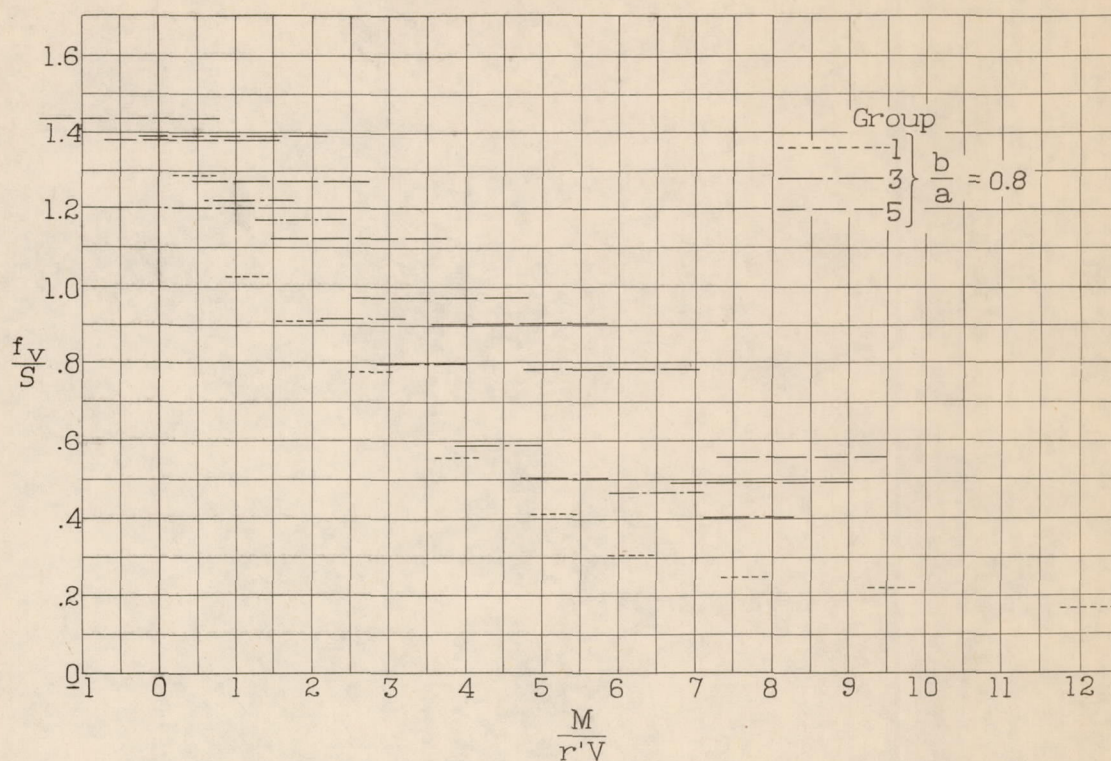
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Figs. 9d,9e,9f

Figure 9d.- Bending-stress diagrams for elliptic cylinders in combined transverse shear and bending.

Figure 9e.- Bending-stress diagrams for elliptic cylinders in combined transverse shear and bending.

Figure 9f.- Bending-stress diagrams for elliptic cylinders in combined transverse shear and bending.

Figure 10a.-Shearing-stress diagrams for elliptic cylinders in com-
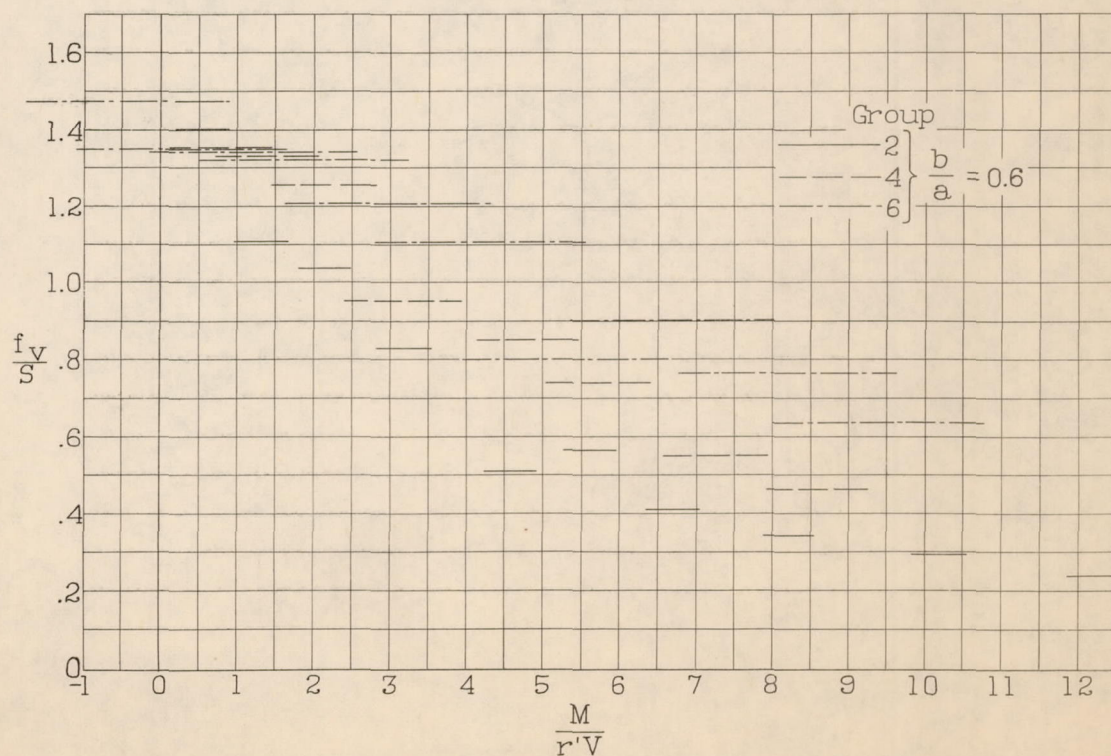bined transverse shear and bending.

Figure 10b.-Shearing-stress diagrams for elliptic cylinders in com-
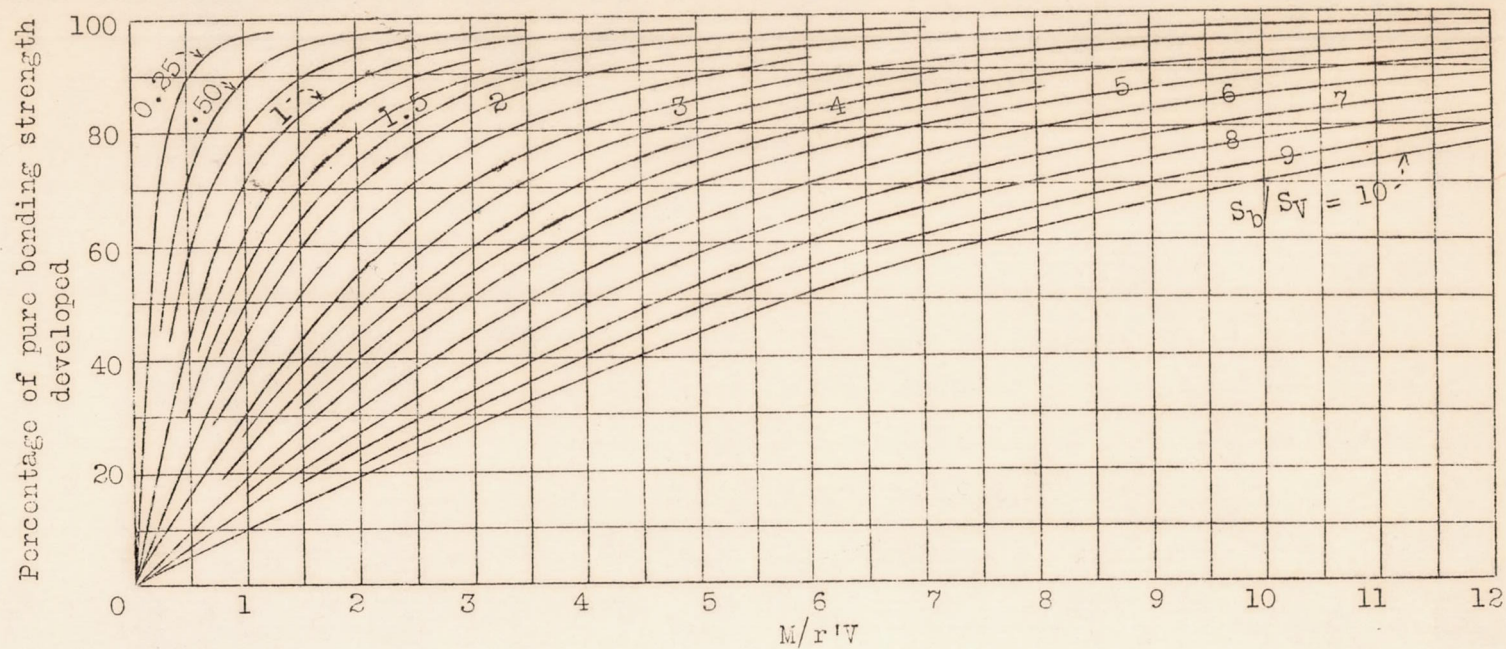bined transverse shear and bending.

Figure 11. – Chart for bending strength of thin-walled cylinders subjected to combined transverse shear and bending.